**Topics: Normal distribution, Functions of Random Variables**

1. **The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
2. **0.3875**
3. **0.2676**
4. **0.5**
5. **0.6987**

**Ans:- 0.2676**

1. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.**
2. **More employees at the processing center are older than 44 than between 38 and 44.**

**Ans: -**

**TRUE**

We have a normal distribution with *μ* = 38 and *σ* = 6. Let X be the number of employees. So according to question

1. Probability of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X ≤ 44).

Z = (X -*μ*)/ = (X - 38)/6

Thus, the question can be answered by using the normal table to find

Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%

So, the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore, the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

1. **A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

**Ans:-**

**TRUE**

Probability of employees less than age of 30 = Pr(X<30).

Z = (X -*μ*)/ = (30 - 38)/6

Thus, the question can be answered by using the normal table to find

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%

So, the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48 (36 employees).

Therefore, the statement B of the question is also TRUE.

1. **If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.**

**Ans: -**

As we know that if X ∼ N (μ1, σ1^2), and Y ∼ N (μ2, σ2^2) are two independent random variables then X + Y ∼ N (μ1 + μ2, σ1^2 + σ2^2), and X − Y ∼ N (μ1 − μ2, σ1^2 + σ2^2).

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).

Therefore, in the question

2X1~ N (2 u,4 σ^2) and X1+X2 ~ N (μ + μ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4μ,6 σ^2)

1. **Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**
2. **90.5, 105.9**
3. **80.2, 119.8**
4. **22, 78**
5. **48.5, 151.5**
6. **90.1, 109.9**

**Ans: -**

D. 48.5, 151.5

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct**.**

1. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**
2. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**

**Ans:-**

Range = 12-19.6×5,12+ 1.96 ×5

=$2.2, $22.8

=Rs.99, Rs.1026

1. **Specify the 5th percentile of profit (in Rupees) for the company**

**Ans:-**P(Z<=(p-12)/5)=0.05

p-12/5= -1.644

p=12 – 8.22 = $3. 78 = rs.170.1

1. **Which of the two divisions has a larger probability of making a loss in a given year?**

**Ans:-**The first division of company, thus have large probability of making Loss in a given year.